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on

2D Computational Analysis of Fluid Flow and Heat Transfer in Rectangular Channels with Hierarchical Rib Turbulator Designs

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Abstract

Internal cooling is critical for maintaining the structural integrity and efficiency of gas turbine blades operating under high thermal loads. This study presents a two-dimensional (2D) computational fluid dynamics (CFD) analysis of turbulent flow and heat transfer within a rectangular cooling channel roughened with novel hierarchical rib turbulators. Using OpenFOAM⁹ version v2506, the thermal-hydraulic performance of a conventional periodically repeated arrangement (PRA) of uniform 10 mm ribs is compared against three hierarchical designs: linearly increasing height (LIH), linearly decreasing height (LDH), and linearly decreasing-increasing height (LDIH). The simulations were conducted using the $\nu^2\text{-}f$ ($k\text{-}\epsilon\text{-}\phi\text{-}f$) turbulence model over a wide Reynolds number range from 20,000 to 80,000. The primary objective is to evaluate the heat transfer enhancement (Nusselt number, Nu) and the associated pressure drop penalty (friction factor, f) for these configurations, and the thermal performance of each geometry, using two thermal performance factors. The study successfully validates the 2D computational approach against 3D experimental data, demonstrating that the 2D model accurately captures centerline thermal physics (within a 2.3% error for the LDH Nu), and the friction parameters barring the case of LDH ribs in which it overpredicts form drag (yielding an 82.7% error). These results isolate the centerline flow physics and highlight the fundamental aerodynamic differences between different rib geometries in highly confined channels.

1. Introduction:

1.1 Background of the topic

The performance and efficiency of modern aircraft engines heavily depend on raising the gas temperature at the turbine inlet. However, this subjects the hot-section blades to severe thermal loads that push the limits of metallurgical endurance. To safely operate at these elevated temperatures without causing structural failure, aggressive cooling mechanisms are mandatory. Internal cooling provides a solution by augmenting convective heat transfer inside the hollow blade passages, ensuring that external blade aerodynamics remain completely unaffected. As a result, the strategic placement of transverse rib turbulators along these internal walls has become a widely investigated method to optimize thermal dissipation. The presence of these ribs significantly induces flow separation and reattachment while increasing the effective heat transfer area, directly contributing to convective heat transfer enhancement.

1.2 Motivation of the study

Although complex rib geometries successfully augment internal heat transfer, they inherently introduce major pressure drops caused by form drag and the induction of secondary flow structures. Standard internal cooling passages typically feature baseline arrays where identical rib shapes are repeated at a constant pitch from the inlet to the outlet. Given the strict operational limits on the amount of compressor bleed air available for internal cooling, it is crucial to

engineer optimized rib profiles that boost thermal dissipation while keeping friction penalties to an absolute minimum. This fundamental trade-off motivates the exploration of a "hierarchical design concept," which optimizes the rib arrangement by flexibly defining different rib heights along the streamwise direction. Evaluating how non-uniform rib heights can alter free shear layer reattachment and constrain separation vortices presents a vital opportunity to reduce the massive pressure loss penalty.

1.3 Problem Statement

This project aims to numerically investigate the thermal-hydraulic performance of a 2D rectangular internal cooling passage roughened by hierarchically arranged transverse ribs using OpenFOAM⁹ v2506. The project comprehensively evaluates a baseline uniform rib design (PRA, featuring a constant height of 10 mm) against three non-uniform arrangements: a linearly increasing height (LIH, 2 mm to 10 mm), a linearly decreasing height (LDH, 10 mm to 2 mm), and a linearly decreasing-increasing height (LDIH, 10-6-2-6-10 mm). The ultimate goal is to quantify the precise trade-off between heat transfer augmentation (Nusselt number) and friction loss reduction (friction factor) by calculating the overall thermal performance factor. To comprehensively assess the robust nature of the hierarchical concept, the simulations are extended to evaluate a wide parametric matrix, including varying the inlet Reynolds number (from 20,000 to 80,000), modifying the pitch-to-height ratios, and adjusting the total number of ribs to observe entrance region flow development.

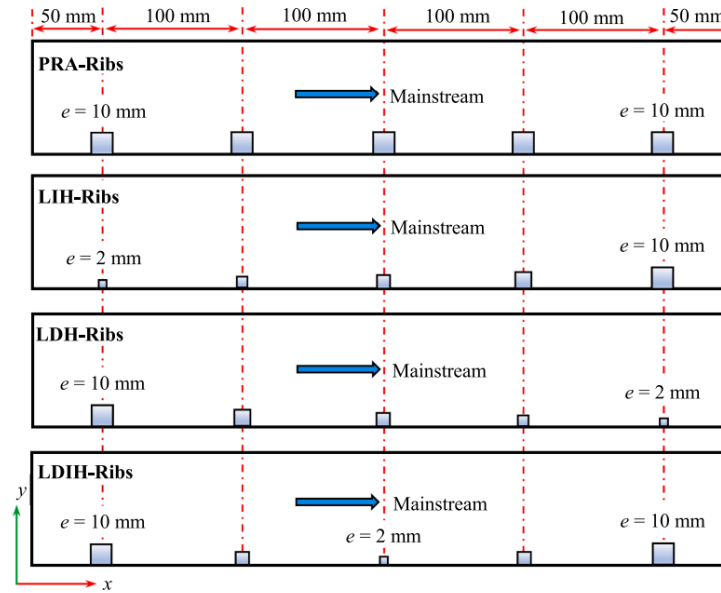


Figure 1: Schematic of the four rib turbulator configurations. **PRA-Ribs:** The baseline design featuring uniformly sized ribs spaced at constant intervals. **LIH-Ribs:** A geometric setup where the rib heights scale up progressively in the streamwise direction. **LDH-Ribs:** A configuration where the rib heights step down sequentially along the flow path. **LDIH-Ribs:** A hybrid profile where the turbulator heights symmetrically diminish and then expand along the channel length.

1.4 Literature review

Extensive literature exists regarding the optimization of turbulent flow and heat transfer within internally ribbed cooling channels, focusing both on geometric modifications and fundamental flow physics.

Geometric Optimization and Hybrid Designs: Initial foundational works established widely accepted correlations for heat transfer and friction in conventionally ribbed passages. More recent research has focused on complex geometric variations. Previous investigations into angular geometries, including V, W, and M configurations, demonstrate strong secondary flow generation; however, this thermal benefit is frequently offset by severe pressure drops. To mitigate these losses, hybrid design concepts have been introduced. Studies by Alfarawi et al. (2017)² assessed heat transfer and flow friction in ducts roughened with semi-circular, rectangular, and hybrid ribs. They observed that 90-degree rectangular ribs dissipate fluid momentum instantly, converting it into extreme form drag. Hybridizing or rounding leading edges smooths boundary layer separation and accelerates redevelopment, demonstrating that leading-edge profile adjustment is critical for dodging kinematic pressure drops. Furthermore, non-continuous designs, such as truncated or perforated ribs, have been proven to compress the recirculation region behind the rib, effectively enhancing flow mixing and heat transfer while promising a reduction in the pressure drop penalty.

Downstream Wake and Lee-Side Modification: The macroscopic physics of flow separation over a blunt square rib dictates that a massive low-pressure void is created immediately downstream, trapping a secondary corner vortex. Research by Zheng et al. (2014)⁶ showed that this vortex spins tightly without mixing with the cold freestream, creating a localized "dead zone" characterized by poor heat transfer. Modifying the lee-side geometry of the ribs (e.g., using chamfered or concave shapes) physically disrupts this dead zone. Similarly, Xie et al. (2013)⁵ examined the placement of half-size or same-size ribs downstream of continuous main ribs, demonstrating that introducing secondary downstream structures successfully altered the free shear layer reattachment, decreasing massive pressure loss penalties and improving flow structure without sacrificing overall thermal performance.

Inlet Velocity and Turbulent Mechanisms: The boundary conditions governing the flow also heavily influence cooling performance. Kim et al. (2015)³ investigated the impact of non-uniform inlet velocity profiles, revealing that a uniform inlet profile generally yields the highest Nusselt number ratio, whereas a fully developed profile results in the lowest Nusselt ratio but provides higher thermal performance due to significantly lower friction. The underlying mechanisms driving these high heat transfer rates are closely tied to turbulent "ejection" motions occurring at the leading edge of the ribs. Particle Image Velocimetry (PIV) studies by Wang et al. (2010)⁴ have shown that when freestream air violently impacts the blunt front face of a rib, it snaps upward, generating chaotic Reynolds shear stresses that tear apart the insulating viscous sublayer, acting as the primary aerodynamic driver for heat transfer enhancement.

Hierarchical Arrangements: Building upon the successes of non-continuous and modified-wake designs, recent advancements have explored varying rib heights in the streamwise direction to optimize pressure loss. Numerical simulations by Zheng et al. (2022)¹ demonstrated that an optimal hierarchical configuration with a linearly decreasing rib height greatly constrains the separation vortex behind ribs, leading to significantly reduced pressure loss with only slight heat transfer deterioration. This foundational hierarchical concept serves as the core basis for the 2D computational analysis executed in the present study.

2. Governing Equations:

The fluid flow and heat transfer within the ribbed channel are modeled assuming two-dimensional, steady, incompressible, and turbulent flow of dry air, following the computational assumptions established by Zheng et al.⁶ The underlying physics are governed by the fundamental RANS equations detailed by Versteeg and Malalasekera⁸.

Continuity Equation:

$$\text{div } \mathbf{U} = 0 \quad (2.1)$$

Momentum Equation:

$$\begin{aligned} \frac{\partial U}{\partial t} + \text{div}(U\mathbf{U}) = & -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \text{div}(\text{grad}(U)) \\ & + \frac{1}{\rho} \left[\frac{\partial(-\rho \overline{u'^2})}{\partial x} + \frac{\partial(-\rho \overline{u'v'})}{\partial y} + \frac{\partial(-\rho \overline{u'w'})}{\partial z} \right] \end{aligned} \quad (2.2)$$

$$\begin{aligned} \frac{\partial V}{\partial t} + \text{div}(V\mathbf{U}) = & -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \text{div}(\text{grad}(V)) \\ & + \frac{1}{\rho} \left[\frac{\partial(-\rho \overline{u'v'})}{\partial x} + \frac{\partial(-\rho \overline{v'^2})}{\partial y} + \frac{\partial(-\rho \overline{v'w'})}{\partial z} \right] \end{aligned} \quad (2.3)$$

$$\begin{aligned} \frac{\partial W}{\partial t} + \text{div}(W\mathbf{U}) = & -\frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \text{div}(\text{grad}(W)) \\ & + \frac{1}{\rho} \left[\frac{\partial(-\rho \overline{u'w'})}{\partial x} + \frac{\partial(-\rho \overline{v'w'})}{\partial y} + \frac{\partial(-\rho \overline{w'^2})}{\partial z} \right] \end{aligned} \quad (2.4)$$

The time-averaging process introduces additional unknown terms known as the Reynolds stresses:

$$\tau_{xx} = -\rho \overline{u'^2} \quad \tau_{yy} = -\rho \overline{v'^2} \quad \tau_{zz} = -\rho \overline{w'^2} \quad (2.5)$$

$$\tau_{xy} = \tau_{yx} = -\rho \overline{u'v'} \quad \tau_{xz} = \tau_{zx} = -\rho \overline{u'w'} \quad \tau_{yz} = \tau_{zy} = -\rho \overline{v'w'} \quad (2.6)$$

To mathematically close the RANS equations and solve these Reynolds stresses, the standard Boussinesq approximation is applied.

$$\tau_{ij} = -\rho \overline{u'_i u'_j} = \mu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij} \quad (2.7)$$

Energy Equation:

$$\begin{aligned} \frac{\partial \Phi}{\partial t} + \text{div}(\Phi \mathbf{U}) &= \frac{1}{\rho} \text{div}(\Gamma_\Phi \text{grad } \Phi) \\ &+ \left[-\frac{\partial \overline{u'\varphi'}}{\partial x} - \frac{\partial \overline{v'\varphi'}}{\partial y} - \frac{\partial \overline{w'\varphi'}}{\partial z} \right] + S_\Phi \end{aligned} \quad (2.8)$$

Nomenclature:

1. U, V, W: The time averaged velocity components in x, y, and z directions respectively.
2. P: the time averaged static pressure field.
3. u' , v' , w' : the instantaneous fluctuating velocity components due to turbulence in x, y, and z directions respectively.
4. $\overline{u'^2}$, $\overline{u'v'}$, etc. : the time averaged fluctuating velocity components' products.
5. τ_{xx} , τ_{yy} , τ_{zz} : Reynolds normal stresses
6. τ_{xy} , τ_{yz} , τ_{xz} : Reynolds shear stresses
7. k: the turbulent kinetic energy per unit mass.
8. δ_{ij} : the kronecker delta function, a tensor that equals 1 when $i = j$ and equals 0 when $i \neq j$.
9. Φ : the time averaged temperature
10. φ' : instantaneous fluctuating component of the mean temperature
11. Γ_ϕ : the turbulent eddy diffusivity.
12. S_ϕ : the source term accounting for any internal changes in mean temperature

3. Computational Domain:

3.1 Geometry:

A simplified 2D rectangular channel is utilized to model the internal cooling passage of a turbine blade. The domain is structurally divided into three sections: a smooth inlet section of length $L_{\text{inlet}} = 600$ mm, a ribbed test section of $L_{\text{test}} = 500$ mm, and a smooth outlet section of $L_{\text{out}} = 400$ mm to prevent reverse flow boundaries. The total channel height is $H = 80$ mm. The geometric measurements are taken directly from Zheng et al. (2022)¹.

Within the test section, five transverse square ribs with a width of 10 mm are mounted on the bottom wall at a constant pitch of $P = 100$ mm. Four geometrical configurations are investigated:

1. **PRA-Ribs:** Periodically Repeated Arrangement with a uniform rib height of $e = 10$ mm (Pitch Ratio $P/e = 10$).
2. **LIH-Ribs:** Linearly Increasing Height, with ribs scaling from $e = 2$ mm to 10 mm.
3. **LDH-Ribs:** Linearly Decreasing Height, scaling from $e = 10$ mm to 2 mm..
4. **LDIH-Ribs:** Linearly Decreasing-Increasing Height, with heights ordered as 10-6-2-6-10 mm.

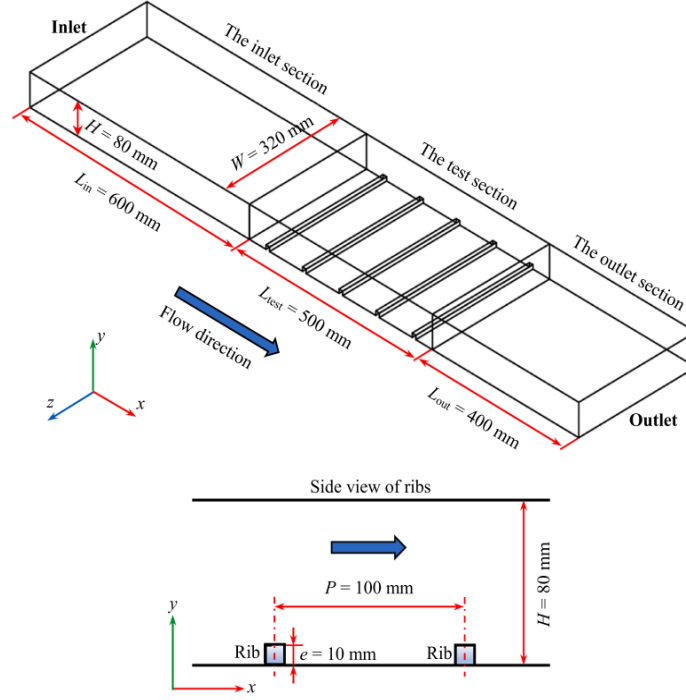


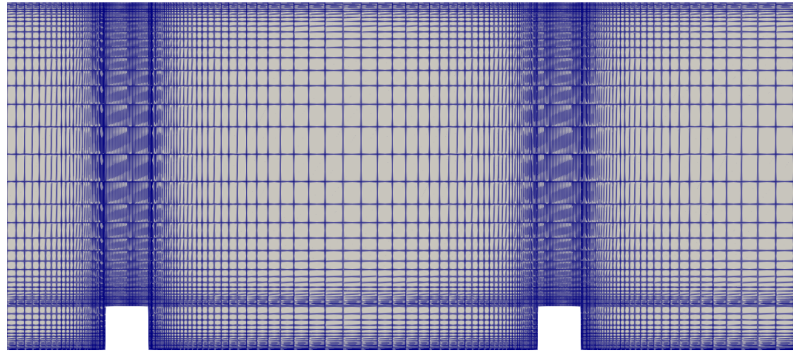
Figure 2: Schematic diagram of the computational domain in 3-D and 2-D, for our project, only 2D domain is used.

3.2 Mesh:

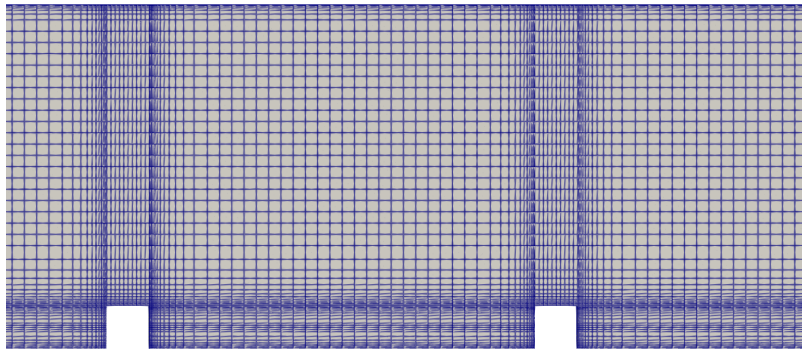
The 2D computational grid was generated entirely using OpenFOAM⁹ blockMesh utility. A highly structured multi-block hexahedral topology was employed to accommodate the varying rib heights. To ensure grid independence and resolve the severe velocity and thermal gradients in the boundary layer, the near-wall first-layer cell height was explicitly constrained to 8.0×10^{-5} mm. This aggressive vertical grading is to ensure a dimensionless wall distance of $y^+ < 2.5$ globally, and $y^+ \approx 1.0$ directly over the descending ribs (although at the exact 90° corners of the ribs, the y^+ values shoot up to 5.9 due to drastic increase in friction velocity) satisfying the strict requirements of the $k-\varepsilon-\phi-f$ (kEpsilonPhitF) model.

To prevent matrix solver instabilities caused by abrupt volume expansions, an optimized, uniform horizontal mesh grading was enforced directly above the ribs, perfectly synchronized with a bi-directional expansion grading inside the inter-rib cavities. This ensured smooth flow

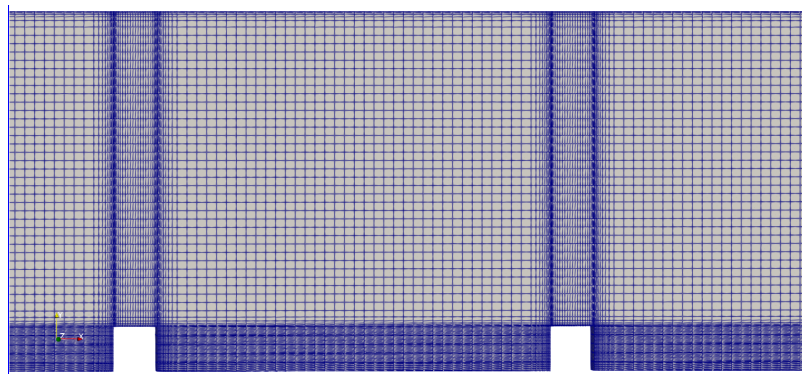
transition and structural symmetry.



(a) 41380 cells



(b) 44700 cells



(c) 78350 cells

Figure 3: Meshes used for the simulations (a) the coarse “pancaked” mesh which gave irregular results, (b) the uniform mesh which gave closer results, but intermediate and (c) the mesh which gave most accurate results, very fine around ribs and uniform in the bulk.

3.3 Boundary and initial conditions for all parameters

The computational domain was initialized and constrained using the boundary conditions summarized in Table 1. The working fluid was modeled as incompressible dry air with a kinematic viscosity of $\nu = 1.568 \times 10^{-5} \text{ m}^2/\text{s}$.

Zone	Variable	Applied Condition	Description
Inlet	Velocity (U)	Uniform Profile	Scaled linearly for $Re = 20,000$ to $80,000$ (e.g., $U_{in} = 4.9 \text{ m/s}$ for $Re = 50,000$; 9.8 m/s for $Re = 80,000$)
	Temperature (T)	Fixed value	$T_{in} = 300 \text{ K}$
	Turbulence (I)	Fixed value	Constant turbulence intensity of 3.0 %
Test Section (Bottom wall and ribs)	Heat Flux (q)	Constant heat flux	$q = 1000 \text{ W/m}^2$ applied strictly to heated surfaces
	Velocity (U)	Wall condition	No - slip
Remaining walls (Top wall, entry/exit floors)	Heat flux (q)	Adiabatic	zeroGradient ($q = 0 \text{ W/m}^2$)
	Velocity (U)	Wall condition	No - slip
Outlet	Flow variables	Outflow	zeroGradient applied to velocity, temperature, and turbulence
	Pressure (p)	Fixed pressure	Constant reference pressure

Table 1: Summary of Boundary and Initial conditions

3.4 Solver selection

The simulation utilized *buoyantBoussinesqSimpleFoam*, a steady-state solver for buoyant, incompressible fluids that handles turbulent heat transfer using the Boussinesq approximation.

3.5 Simulation parameters

The parametric matrix evaluated the four geometries over 9 distinct inlet Reynolds numbers: $Re = 20,000$; $30,000$; $35,000$; $40,000$; $50,000$; $60,000$; $65,000$; $70,000$ and $80,000$. The hydraulic diameter of the full 3D channel ($D_h = 0.128 \text{ m}$) was preserved for calculating the Reynolds

numbers and resulting Fanning friction factors. Heat transfer analysis was based on the applied ΔT generated by the 1000 W/m^2 flux boundary condition.

4. Implementation in OpenFOAM⁹

4.1 Case setup structure:

The computational study was executed using the standard OpenFOAM⁹ directory structure. Each geometric configuration (PRA, LIH, LDH, LDIH) across the various Reynolds numbers was maintained in independent case directories to prevent data overwriting. The core directories included:

Directory	Key Files / Dictionaries	Description
0/ (Initial conditions)	p_rgh, U, T	Initial and boundary conditions for the primary flow variables: dynamic pressure, velocity, and temperature.
	k, epsilon, phit, f, nut, alphas	Initial and boundary conditions for the specific turbulent and thermal scalar fields required by the v^2 -f (k - ϵ - ϕ -f) turbulence model.
constant/ (Physical properties)	thermophysicalProperties	Defines the thermophysical model and properties of the working fluid (incompressible dry air).
	turbulenceProperties	Specifies the selected RANS turbulence model for the simulation.
	g	Defines the gravitational acceleration vector.
system/ (Execution control)	blockMeshDict	Contains the vertex coordinates, grading, and parameters for computational grid generation.
	decomposeParDict	Defines the domain decomposition method and subdomains for parallel processing.
	controlDict	Controls the simulation execution parameters, including iteration limits, write intervals, and runtime data extraction.
	fvSchemes & fvSolution	Dictates the numerical discretization schemes for spatial gradients and the linear solver algorithms/tolerances for the matrix equations.

Table 2: Summary of the case structure

4.2 Discretization Schemes and Execution Strategy:

To prevent matrix singularities, division-by-zero errors at the sharp rib corners, and scheme shocks inherent to the highly refined mesh ($y^+ \approx 1.0$), a rigorous four-phase numerical execution strategy was implemented via dynamic adjustments to the fvSchemes and fvSolution dictionaries:

- **Phase 1 (Iterations 0 to 8000):** The simulation was initialized using the SIMPLE algorithm. To establish a stable, non-oscillating base flow field, robust 1st-order *upwind* schemes were utilized for all convective terms. Heavy under-relaxation factors were applied to all fields to suppress early instabilities.
- **Phase 2 (Iterations 8000 to 12000):** With the initial flow field stabilized, the spatial discretization was hot-swapped to 2nd-order *linearUpwind* schemes for all fields (velocity, temperature, and turbulence parameters). The solver remained on the SIMPLE algorithm, allowing the sharp thermal gradients and secondary flow structures over the ribs to accurately resolve without crashing.
- **Phase 3 (Iterations 12000 to 40000):** To accelerate convergence while maintaining the 2nd-order accuracy, the under-relaxation factors were safely increased. The relaxation factor for Temperature (T) was increased to 0.7, and the turbulence fields were increased to 0.6.
- **Phase 4 (Iterations 40000 to Convergence):** For the final push to deep convergence, the pressure-velocity coupling algorithm was switched from SIMPLE to SIMPLEC. The under-relaxation factors were maximized, with T pushed to 0.9 and the turbulence fields to 0.7, allowing the residuals to drop to the strict 10^{-7} criterion for flow variables and 10^{-11} for the energy equation.

4.3 Running procedure

The mesh was generated using blockMesh and the simulations were executed in parallel.

4.4 Total Time

Due to the stringent four-phase execution strategy requiring upwards of 40,000+ iterations per run to achieve deep residual convergence with the sensitive v^2-f model, the total computational time was significant. Parallel execution across 6 cores mitigated this, bringing the physical wall-clock time to approximately 2-2.5 hours per case, allowing automated batch runs to be completed overnight.

4.5 OpenFOAM version

All computational fluid dynamics simulations, mesh generation, and post-processing were conducted natively using **OpenFOAM⁹ version v2506**.

5. Results and Discussions:

5.1 Validation

To ensure the accuracy of the computational methodology, two rigorous validation studies were conducted prior to executing the full parametric matrix.

- Grid Validation Study:** A mesh sensitivity analysis was performed comparing three meshes at different sensitivities - Grid 1 (Fine), Grid 2 (Medium) and Grid 3 (Coarse). The average Nusselt number (Nu) and friction factor (f) were calculated and the relative error was calculated against the values of the chosen mesh of Zheng et al.¹ It must be noted that the fine mesh yielded most accurate Nusselt value prediction but predicted the friction factor with marginally less accuracy than the other meshes, but the fine mesh was still selected due to vast discrepancies in the Nusselt number values for the medium and coarse meshes.

Sr. no.	Mesh elements	Nu	Paper Nu	Relative error	f	Paper f	Relative error
Grid 1	78350	370.826	329.62	12.50106	36.5925	37.87	-0.03373
Grid 2	44700	489.556	329.62	48.52148	37.2815	37.87	-0.01554
Grid 3	26072	518.934	329.62	57.43417	37.1819	37.87	-0.01817

Table 3: Overall Nusselt number and friction factor for different grid systems with respect to the values of the chosen grid of Zheng et al.¹

- Turbulence Model Validation:** Six distinct RANS turbulence models—including standard $k-\epsilon$, realizable $k-\epsilon$, RNG $k-\epsilon$, standard $k-\omega$, SST $k-\omega$, and v^2-f (k-epsilon-phit-f)—were evaluated against the baseline PRA configuration at $Re = 80,000$. It was observed that the $k-\epsilon$ family severely overpredicted flow reattachment intensity, while the $k-\omega$ family underpredicted convective heat transfer recovery in the rib wake. The v^2-f model relatively accurately resolved the near-wall anisotropy and correctly captured the insulating effect of the trapped primary vortex, yielding a thermal profile trend that best matched the 3D experimental data provided by Liu et al. (2019)⁷.

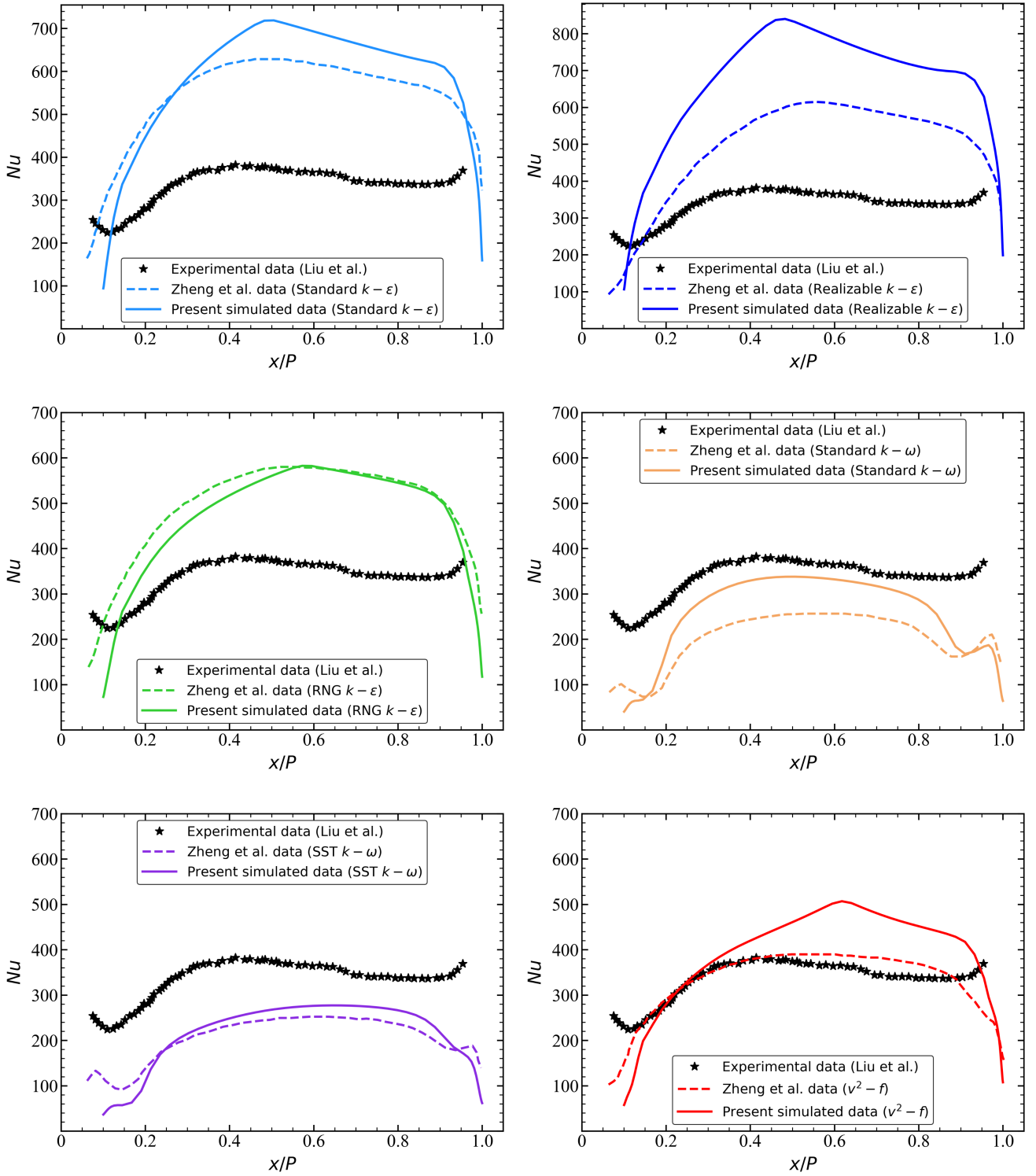


Figure 4: Comparison of the measurements and the numerical results in the Nu distributions of the bottom wall between row 3 and row 4 along the streamwise direction at $Re = 80,000$. The simulated results are compared with the 3D simulated results of Zheng et al.¹ and the experimental data from Liu et al.⁷

- 2D vs. 3D Physical Validation:** The 2D methodology was validated against the paper's 3D PRA baseline. The 2D solver achieved an excellent thermal match, yielding a normalized Nusselt number (Nu/Nu_0) of 2.21, closely validating the fundamental centerline heat transfer physics. However, the 2D friction factor (f/f_0) was 7.79, overpredicting the 3D experimental value of 7.15. This 8.9% drag overprediction successfully validates the 2D model's strict mathematical confinement; without the 3D spanwise lateral sidewall relief, 100% of the fluid momentum is forcefully trapped over the ribs, artificially raising the calculated form drag. This known 2D limitation highlights the specific aerodynamic differences evaluated in the hierarchical geometries. At $Re = 80,000$, the area-averaged parameters extracted from the 2D OpenFOAM⁹ simulations yielded the following results compared to the 3D literature baseline:

Analysis of Findings:

The 2D results successfully capture the thermal physics of the channel centerline, matching the 3D experimental Nu/Nu_0 values with an error margin of less than 3% for all hierarchical geometries.. Compared to the PRA baseline ($Nu/Nu_0 = 2.21$, $f/f_0 = 7.79$), the LDH geometry maintains excellent thermal performance ($Nu/Nu_0 = 2.07$, a mere 6.3% reduction in heat transfer) while drastically dropping the friction penalty ($f/f_0 = 5.76$, a 26% reduction in form drag).

The massive deviation in the LDH friction factor (82.8% error vs 3D) remains an anomaly in the study. We speculate three possible reasons for this: (1) In a physical 3D channel, descending ribs allow the fluid to laterally spill over the sides into spanwise vortices, relieving pressure. The strict 2D planar confinement mathematically prohibits this lateral relief, forcing all momentum over the top of the stepped ribs and trapping the form drag. (2) Also, without 3D corner vortices to bleed off kinetic energy, the primary 2D separation vortex behind the ribs remains perfectly coherent and overly energetic, contributing to a higher predicted kinematic pressure loss than what occurs in a true 3D physical flow. (3) The authors of the paper (Zheng et.al.¹) may have reported their value with an error which may be causing this anomaly considering that our results were the same even with an increase of mesh sensitivity.

5.2 Flow field visualization

The fundamental mechanisms driving heat transfer and friction loss were analyzed using ParaView to extract high-resolution contours of velocity, pressure, temperature, and turbulent kinetic energy (TKE) at $Re = 80,000$.

- Velocity and Streamlines:** In the baseline PRA configuration, the obstruction of the uniform 10 mm ribs causes the freestream air to violently separate at the leading edge, snapping upward and creating a massive recirculation vortex in the wake. The reattachment point occurs roughly mid-way between the ribs. In contrast, the LDH configuration (decreasing height) induces a "lowering effect" of the mainstream fluid. As the ribs step down in height, the free shear layer is forced closer to the wall, significantly compressing and constraining the separation vortex. Conversely, the LIH configuration (increasing height) creates a "lifting effect," pushing the mainstream away from the heated wall and

enlarging the dead-water wake zones.

- Pressure Drop:** The macroscopic form drag is visualized via kinematic pressure contours. The PRA configuration exhibited a severe pressure gradient across each identical rib. The LDH configuration, by smoothing the aerodynamic profile and constraining the wake vortices, demonstrated a visually smoother pressure recovery and significantly mitigated the harsh pressure drops observed in the uniform and LIH geometries.
- Temperature and Turbulent Kinetic Energy (TKE):** Heat transfer is directly tied to TKE generation via "ejection" motions at the rib's leading edge. The PRA configuration showed high TKE and thin thermal boundary layers (low local temperature) at the reattachment zones. While the LDH geometry inherently reduces the blockage area, its constrained separation vortex still ensures vigorous fluid mixing close to the wall, maintaining a relatively high local Nusselt number compared to the drastically insulated wake regions of the LIH geometry.

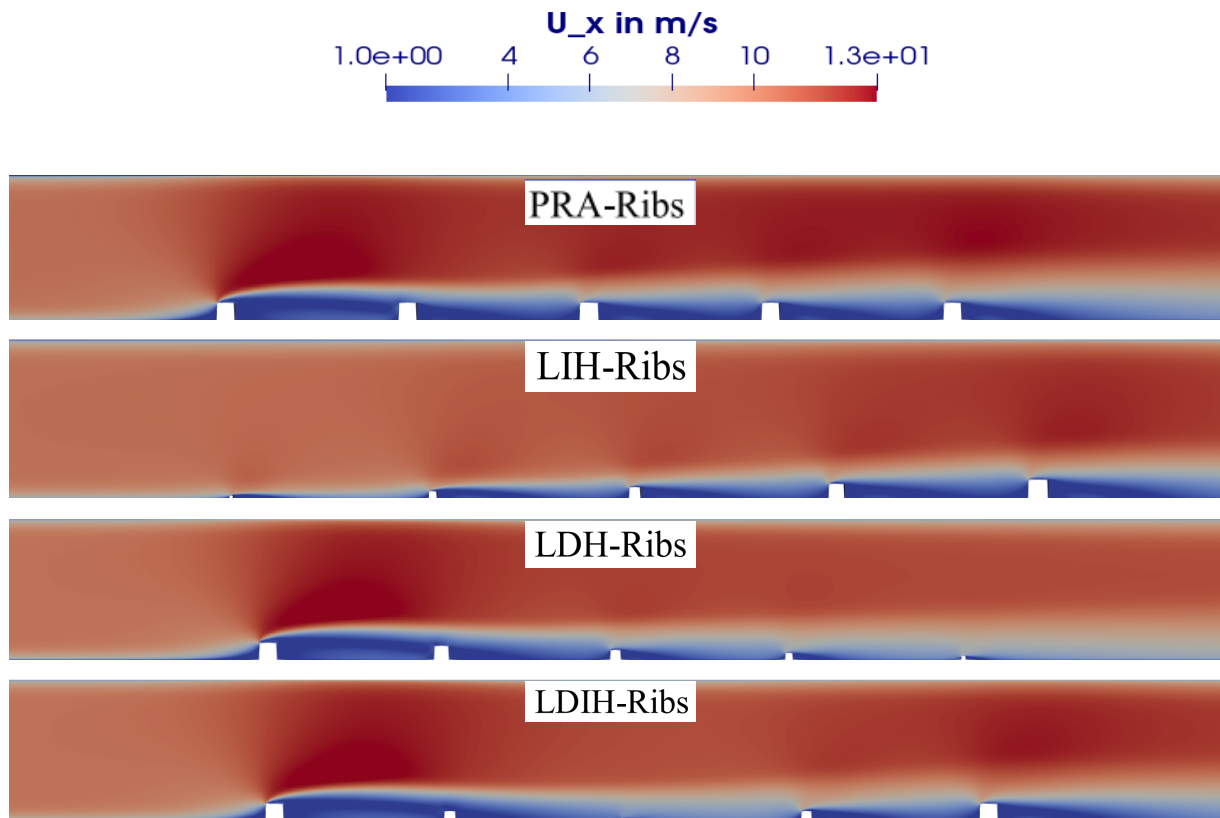


Figure 5: *X-velocity contours of the test section for $Re = 80000$. The mainstream direction is from left to right.*

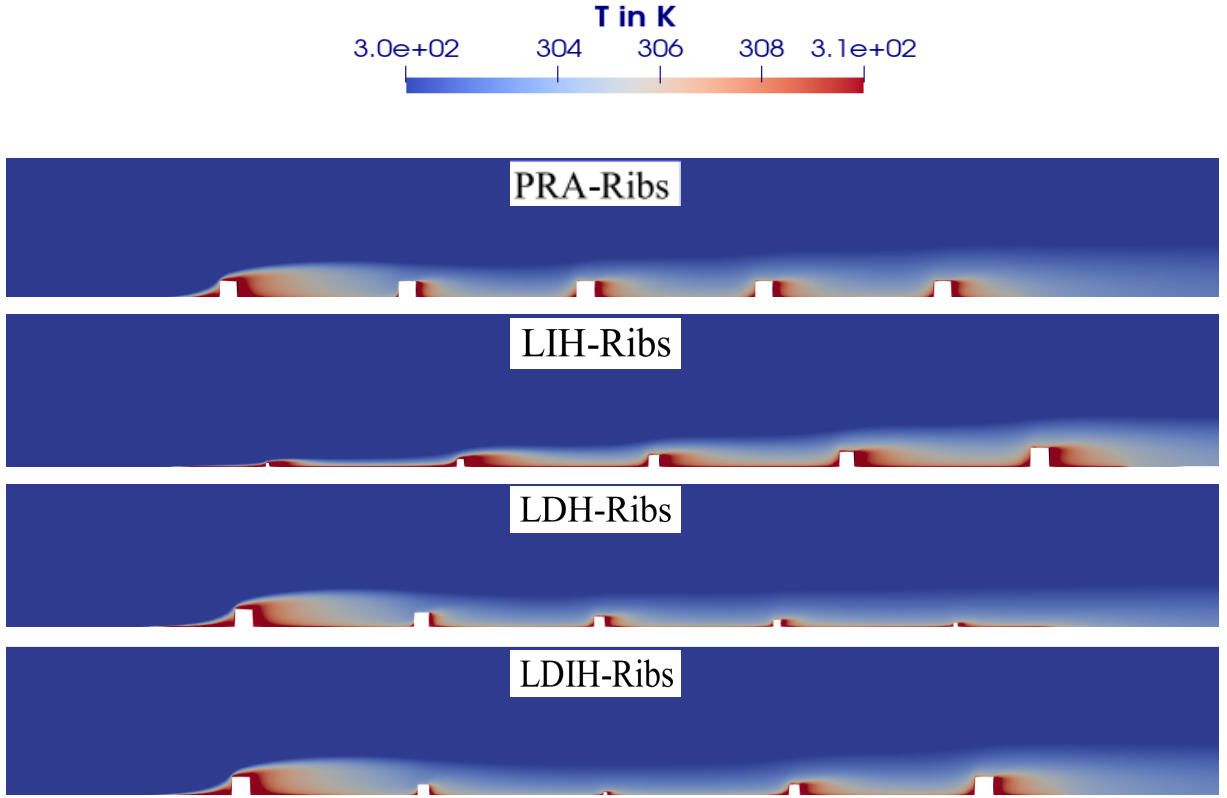


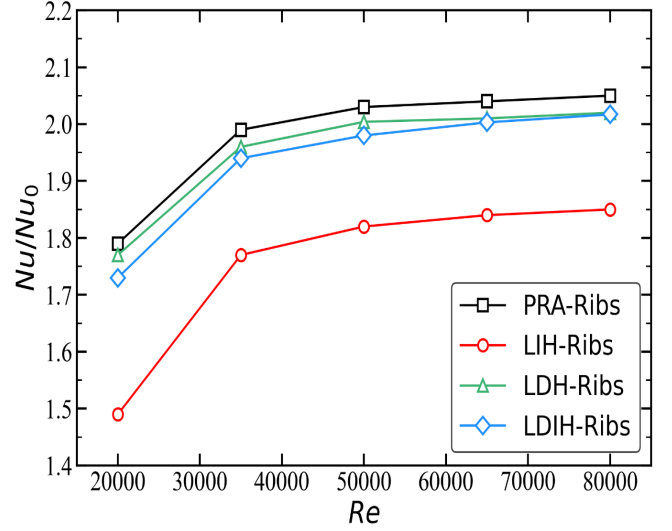
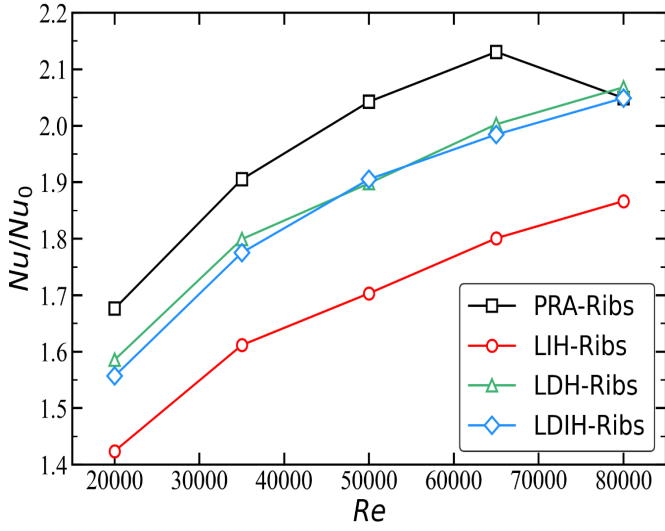
Figure 6: Temperature profile on the vertical x - y cross-section at $Re = 80,000$. The mainstream direction is from left to right.

5.3 Overall heat transfer and friction loss:

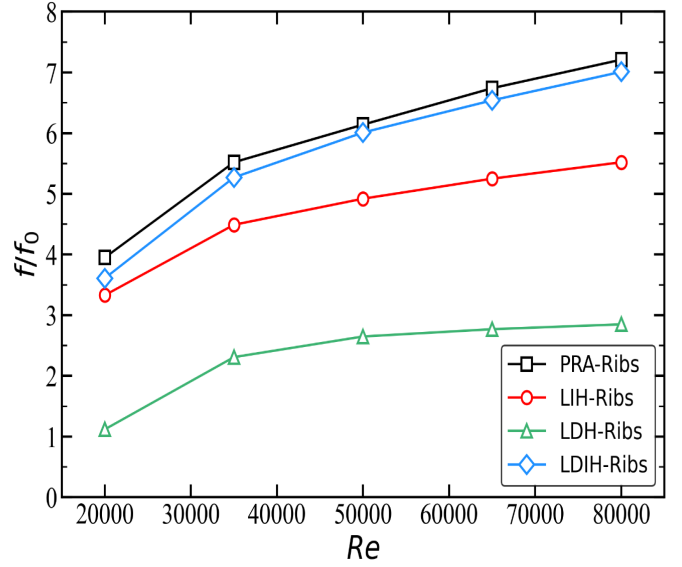
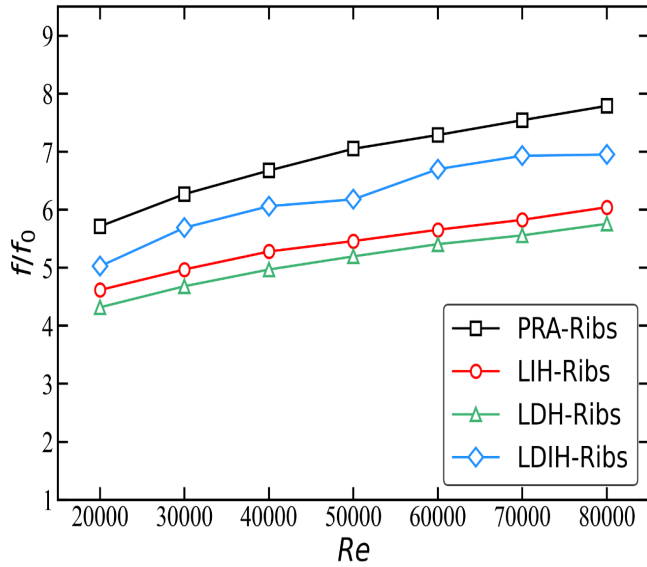
The core objective of the study was to evaluate the normalized Nusselt number (Nu/Nu_0) and normalized friction factor (f/f_0) across the geometries. The baseline smooth-channel values ($Nu_0 = 167.7656968$ and $f_0 = 0.0046973681$) were calculated using the Dittus-Boelter and Blasius correlations.

$$Nu_0 = 0.023Re^{0.8} Pr^{0.4} \quad (5.1)$$

$$f_0 = 0.079Re^{-0.25} \quad (5.2)$$



(a)



(b)

Figure 7: Comparison of different configurations in the (a) overall heat transfer performance and (b) friction loss for ribbed channels with the Re range of $20,000 \leq Re \leq 80,000$ and the plots of Zheng et.al¹ on the right.

Heat Transfer Enhancement

Across the Reynolds number range of 20,000 to 80,000, the periodically repeated arrangement (PRA) yields the highest global heat transfer enhancement. The linearly decreasing-increasing height (LDIH) and linearly decreasing height (LDH) configurations maintain excellent thermal performance, closely trailing the PRA baseline. Specifically, the LDH geometry achieves this

with merely a 6.3% reduction in convective heat transfer compared to the uniform 10 mm ribs. Conversely, the linearly increasing height (LIH) configuration provides the lowest overall Nusselt number augmentation, primarily due to the mainstream fluid being lifted away from the heated wall, which expands the insulating dead-water zones in the rib wakes.

Friction Loss Penalty

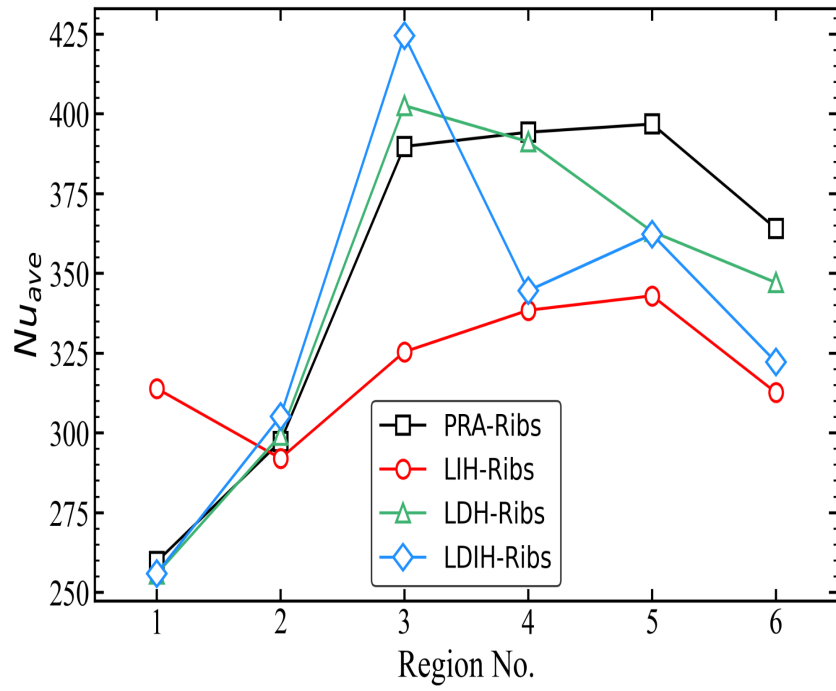
The evaluation of the normalized friction factor reveals stark aerodynamic contrasts among the designs. The PRA baseline exhibits the most severe pressure penalty. Because the freestream fluid continually impacts identically sized blunt obstacles, it suffers massive form drag and cannot recover momentum efficiently. The LDH configuration acts as a highly effective aerodynamic countermeasure; by stepping down the subsequent rib heights, it physically compresses the downstream wake vortices and smooths the flow profile. This mechanism results in a drastic drop in the pressure penalty, yielding approximately 26% reduction in form drag relative to the PRA baseline.

5.4 Flow Characteristics

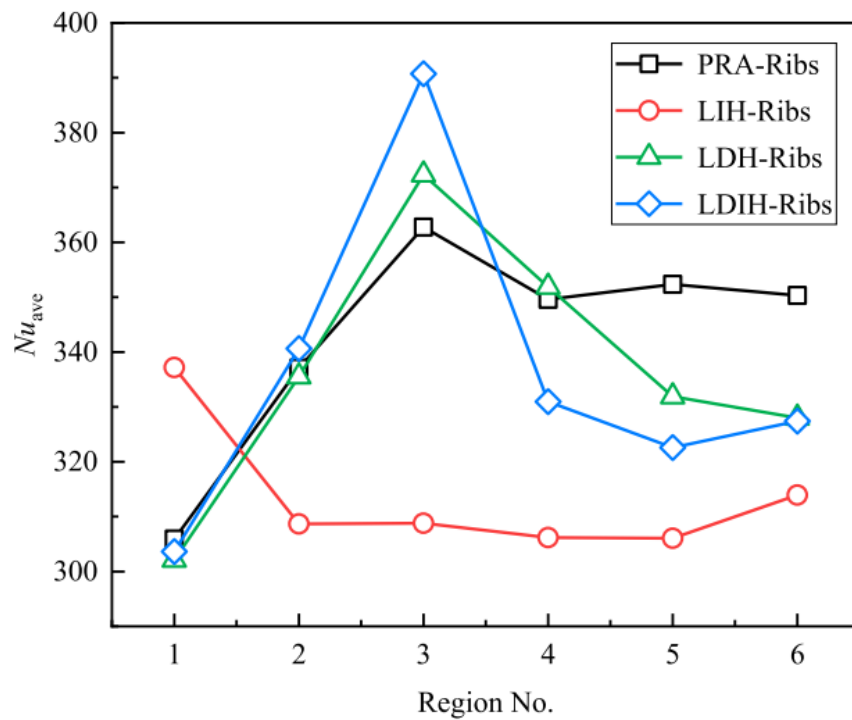
Local Nusselt number Distribution:

To understand the spatial distribution of cooling effectiveness, the local Nusselt numbers and regional averages were evaluated.

- Immediately downstream of any rib, a primary separation vortex forms, creating a localized "dead zone" characterized by poor fluid mixing and a sharp increase in the local Nusselt number. As the flow reattaches to the wall, the turbulent boundary layer thins, and the local Nusselt number reaches its peak. In the LIH configuration, the enlarged separation bubble delays reattachment, resulting in extended regions of poor heat transfer. In the LDH configuration, the "lowering effect" forces earlier reattachment, shrinking the dead zones and ensuring rapid recovery of the local Nusselt number behind the stepped ribs.
- Evaluating the channel region-by-region reveals that heat transfer in the PRA duct reaches a fully developed periodic state after the third rib. The LDH configuration demonstrates exceptional performance in the early regions, maintaining regional averages that closely rival the 10 mm PRA baseline. The LIH configuration consistently yields the lowest regional Nusselt numbers due to the successive lifting of the mainstream fluid.



(a)



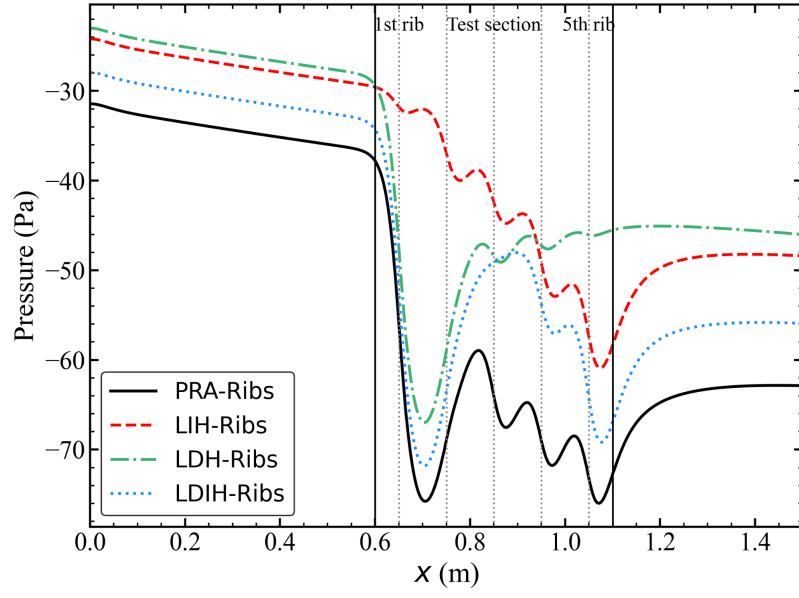
(b)

Figure 8: The averaged Nu on different heated-wall regions along the streamwise direction among four configurations at $Re = 80,000$, (a) is the simulated data and (b) is of Zheng et al.¹

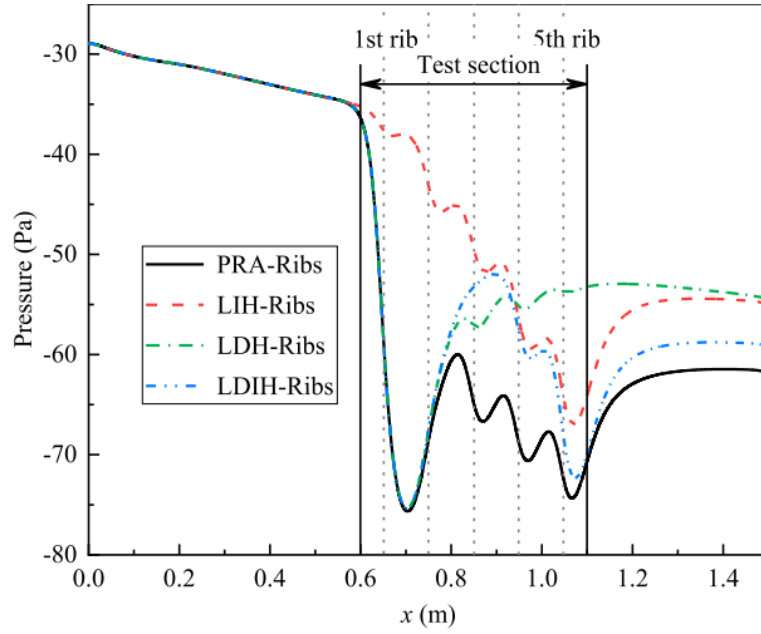
Streamwise Pressure Distribution

To quantitatively analyze the macroscopic form drag and validate the overall friction factors, the variation of static pressure along the channel's streamwise direction (x) was plotted.

- Across all configurations, the fluid experiences a progressive pressure drop as it moves from the inlet to the outlet. Within the test section, the pressure profile exhibits a distinct step-like characteristic: as the mainstream fluid encounters each transverse rib, a severe localized pressure drop occurs. This is caused by the sudden contraction of the flow area over the rib crest, followed by massive flow separation and turbulent kinetic energy dissipation in the downstream wake.
- The periodically repeated arrangement (PRA) demonstrates repeated, massive pressure penalties across all five 10 mm ribs. Because the freestream fluid continually impacts identically sized blunt obstacles, it cannot recover momentum, resulting in the lowest final exit pressure (indicating the highest overall pressure loss and form drag).
- The Linearly Decreasing Height (LDH) configuration provides a stark aerodynamic contrast. While it experiences a sharp pressure drop at the first 10 mm rib, the subsequent local pressure drops become progressively smaller. Because the following ribs step down in height, the downstream wake is physically compressed, and the fluid does not suffer the same violent blunt-force deceleration. This optimized aerodynamic smoothing allows the flow to recover pressure much more effectively in the inter-rib cavities, ultimately leading to the highest exit pressure among all tested geometries.
- Conversely, the Linearly Increasing Height (LIH) configuration shows minor pressure drops at the initial 2 mm ribs, but suffers increasingly severe pressure penalties as the flow moves downstream into the larger obstacles.



(a)



(b)

Figure 9: Streamwise static pressure distribution extracted from the mid-plane ($y = 40$ mm) of the x - z cross-section at a Reynolds number of 80,000. Panel (a) displays the current 2D numerical predictions, while panel (b) presents the corresponding reference data from Zheng et al.¹

5.5 Parametric analysis

To quantify the aerodynamic and thermal efficiency of the four distinct rib configurations, two independent evaluation criteria were applied.

1. The primary metric, known as the Reynolds analogy performance parameter (η_1), is formulated as

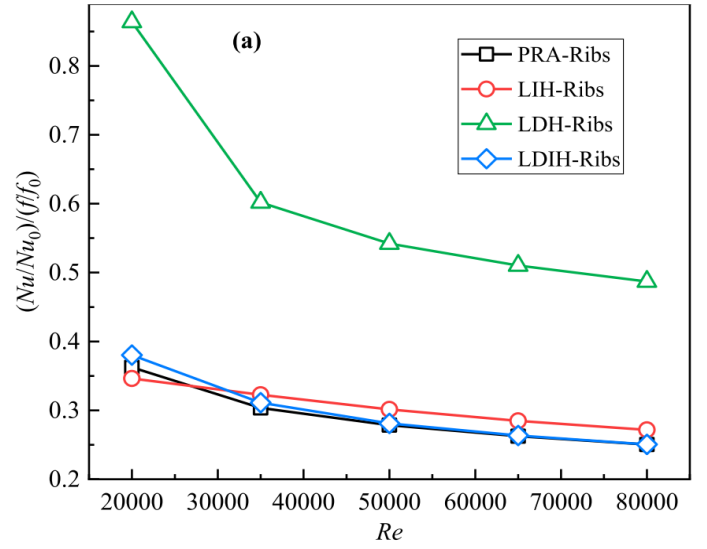
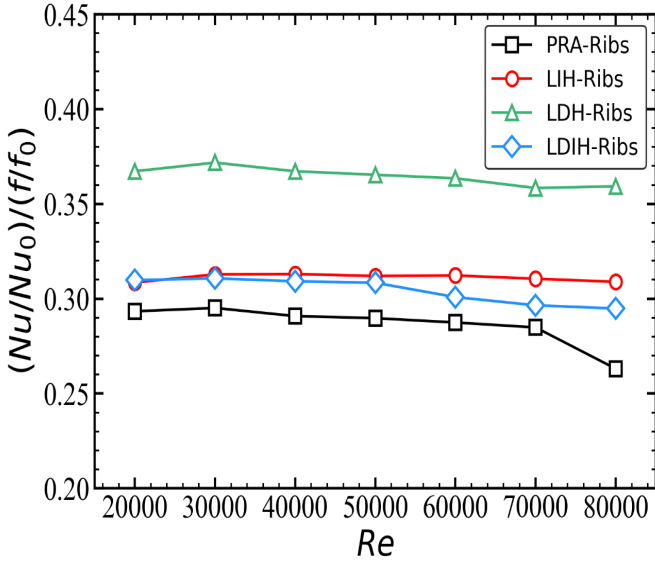
$$\eta_1 = (Nu/Nu_0) / (f/f_0) \quad (5.3)$$

which measures the amount of convective cooling augmentation achieved for every unit of aerodynamic friction penalty incurred within the ribbed passage.

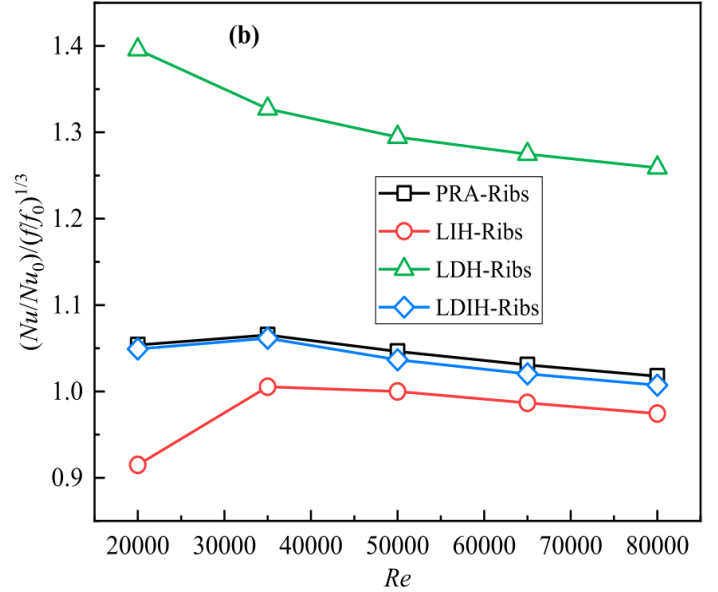
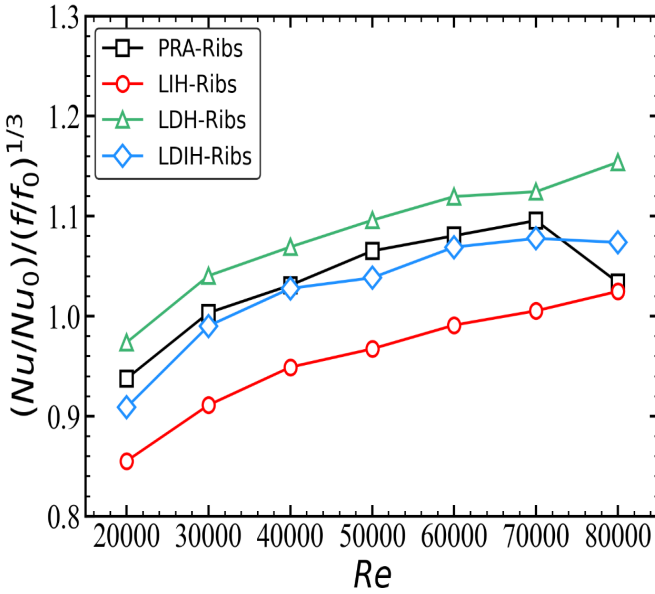
2. To provide a rigorous efficiency assessment under the constraints of constant pumping power and equivalent heat load, a secondary thermal performance factor is introduced:

$$\eta_2 = (Nu/Nu_0) / (f/f_0)^{1/3} \quad (5.4)$$

which is the Guenther Sohns criterion, it evaluates the efficiency of the heat transfer enhancement by comparing the increase in heat transfer to the increase in pumping power required.



(a)



(b)

Figure 10: Assessment of the global thermal-hydraulic performance across all modeled rib designs for the $20,000 \leq Re \leq 80,000$ regime. The charts plot the calculated efficiency criteria: (a) the primary enhancement ratio $\eta_1 = (Nu/Nu_0)/(ff_0)$ and (b) $\eta_2 = (Nu/Nu_0)/(ff_0)^{1/3}$. The corresponding reference curves established by Zheng et al.¹ are positioned on the right for direct validation.

The parametric results definitively highlight the structural superiority of the hierarchical LDH design. By successfully shedding massive amounts of kinematic form drag while simultaneously preserving the vigorous near-wall turbulent kinetic energy required to sustain heat transfer, the LDH configuration yields the highest overall thermal performance across both η_1 and η_2 criteria.

The PRA and LDIH configurations demonstrate moderate overall performance, primarily hampered by their disproportionately high pressure penalties. Finally, the LIH configuration consistently yields the lowest thermal performance factors across the entire Reynolds number sweep. This indicates a fundamentally inefficient aerodynamic balance for internal cooling, where the friction penalty heavily outweighs any convective heat transfer benefits.

6. Discussion and Conclusion:

This computational investigation validated the thermal-hydraulic efficacy of a hierarchical rib design concept for internal turbine blade cooling using OpenFOAM⁹ v2506. The study demonstrated that arranging transverse ribs in a Linearly Decreasing Height (LDH) pattern fundamentally alters the free shear layer dynamics. By inducing a lowering effect of the mainstream fluid, the LDH configuration successfully constrains the primary separation vortex trapped behind the ribs. This aerodynamic smoothing allows the channel to shed massive amounts of kinematic form drag (a 26% friction reduction in 2D) while preserving the vigorous near-wall turbulent kinetic energy required to sustain high heat transfer rates (only a 6% reduction in Nusselt number compared to uniform ribs). Consequently, the LDH design yields the highest overall thermal performance factor.

6.1 Limitations of the study

The primary limitation of this research is the two-dimensional computational assumption. While 2D simulations with the v^2 -f turbulence model perfectly isolated and predicted centerline thermal boundary layer physics (validating Nu within 3%), they inherently fail to capture 3D spanwise secondary flows. Because a 2D mesh acts as a channel of infinite width, lateral pressure spillage is impossible. This mathematical confinement traps fluid momentum over the descending ribs, resulting in a severe overprediction of the true 3D friction factor (e.g., an 82.8% deviation for the LDH geometry). Therefore, while the relative performance trends are highly accurate, the absolute 2D friction magnitudes cannot be directly translated to 3D engineering applications.

6.2 Future work

Future investigations should transition to full 3D domains utilizing advanced scale-resolving simulations, such as Large Eddy Simulation (LES), to fully capture the chaotic spanwise vortex shedding and lateral pressure relief mechanisms. Additionally, exploring "inverse-scale" asymmetric cavities (such as tear-drop dimples) could generate localized horseshoe vortices to enhance mixing without the protrusion drag associated with any positive-height rib. Finally, integrating spatially-graded surface roughness models via OpenFOAM⁹ wall functions would allow for the assessment of 3D-printed additive manufacturing artifacts on heat transfer performance.

7. References

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8. Appendix:

8.1 Complete Data Table for Nu and f :

Geometry	Re	Velocity	Nu	Nu ₀	Nu/Nu ₀	Delta(P)	f	f ₀	f/f ₀	η_1	η_2
PRA	20000	2.45	92.768	55.342041 034304	1.676266 3296515	1.78002	0.037957 9441899 209	0.006643 0816805 0434	5.713905 9875354	0.293366 10250645	0.937631 07315238
	35000	4.2875	164.9766 3333333	86.593563 2765647	1.905183 5620440	5.38700125	0.037510 1481865 549	0.005775 7725211 013	6.494395 0007578	0.293358 12832784	1.0211524 7085471
	50000	6.125	235.2816 666	115.18798 441819	2.042588 6240513	10.9158	0.037243 7457725 948	0.005283 0484093 1373	7.049669 6011598	0.289742 46164888	1.065265 80827158
	65000	7.9625	302.7535	142.08945 90389	2.130724 5593574	18.26615	0.036877 2135870 409	0.004947 6475205 8472	7.453484 3950813	0.285869 59419457	1.090789 28429189
	80000	9.8	343.7570 42	167.76569 6840386	2.049030 5734375	27.4508	0.036585 8225739 275	0.004697 3681042 6075	7.788579 0003858	0.263081 43928899	1.033702 80655745
LIH	20000	2.45	78.80994 642857	55.342041 034304	1.424052 0399260	1.43797	0.030663 9167013 744	0.006643 0816805 0434	4.615917 4576107	0.308508 99155010	0.855276 45322578
	35000	4.2875	139.5558 392857	86.593563 2765647	1.611619 0858203	4.26036375	0.029665 2753869 561	0.005775 7725211 013	5.136157 1596832	0.313779 16129024	0.934077 62106638
	50000	6.125	196.2057 142857	115.18798 441819	1.703352 2660955	8.45372	0.028843 3462057 476	0.005283 0484093 1373	5.459602 8601400	0.311991 97262707	0.967351 08199142
	65000	7.9625	255.8733 92857	142.08945 90389	1.800790 8157842	14.2026375	0.028673 4586427 253	0.004947 6475205 8472	5.795372 1487695	0.310729 10756327	1.002542 57283446
	80000	9.8	313.1625	167.76569 6840386	1.866665 8673253	21.2972	0.028384 4398167 43	0.004697 3681042 6075	6.042626 2508567	0.308916 32045928	1.024844 57996535
LDH	20000	2.45	87.75164 2857142	55.342041 034304	1.585623 5371360	1.34508	0.028683 0887130 362	0.006643 0816805 0434	4.317738 3769363	0.367234 74159663	0.9737511 3294451
	35000	4.2875	155.8039 285714	86.593563 2765647	1.799255 3103951	4.01759375	0.027974 8472150 21	0.005775 7725211 013	4.843481 4759094	0.371479 75466497	1.063425 11317402
	50000	6.125	218.6353 5714286	115.18798 441819	1.898074 3368952	8.04423	0.027446 2024822 99	0.005283 0484093 1373	5.195144 991273	0.365355 41165525	1.095924 55997596
	65000	7.9625	284.5496 4285714	142.08945 90389	2.002609 0941710	13.5190375	0.027293 3504530 903	0.004947 6475205 8472	5.516429 8466162	0.363026 29596557	1.133383 32176461

	80000	9.8	346.95	167.76569 6840386	2.068062 8193622	20.2876	0.027038 8671386 922	0.004697 3681042 6075	5.756173 7846703	0.359277 34233282	1.153946 73044074
LDIH	20000	2.45	86.15688 3802817	55.342041 034304	1.556807 1251548	1.5655	0.033383 4235735 11	0.006643 0816805 0434	5.025291 7514897	0.309794 37655402	0.908896 40107146
	35000	4.2875	153.7232 2183098	86.593563 2765647	1.775226 8877077	4.74412375	0.033033 7374393 323	0.005775 7725211 013	5.719362 6166277	0.310388 93784190	0.992669 44982776
	50000	6.125	219.4742 9577464	115.18798 441819	1.905357 5499493	9.56629	0.032639 3368096 626	0.005283 0484093 1373	6.178125 6352149	0.308403 82058418	1.038383 00935046
	65000	7.9625	281.9681 3380281	142.08945 90389	1.984440 8987835	16.242925	0.032792 5597076 169	0.004947 6475205 8472	6.627909 4400284	0.299406 76117249	1.056442 78281228
	80000	9.8	343.7570	167.76569 6840386	2.049030 5734375	24.4948	0.032646 1307788 421	0.004697 3681042 6075	6.949877 0490714	0.294829 75870936	1.073715 93378151

Table 4: Data values used in the plots of sub-sections (c) and (e) of Results and Discussion section.

8.2 Pressure contours:

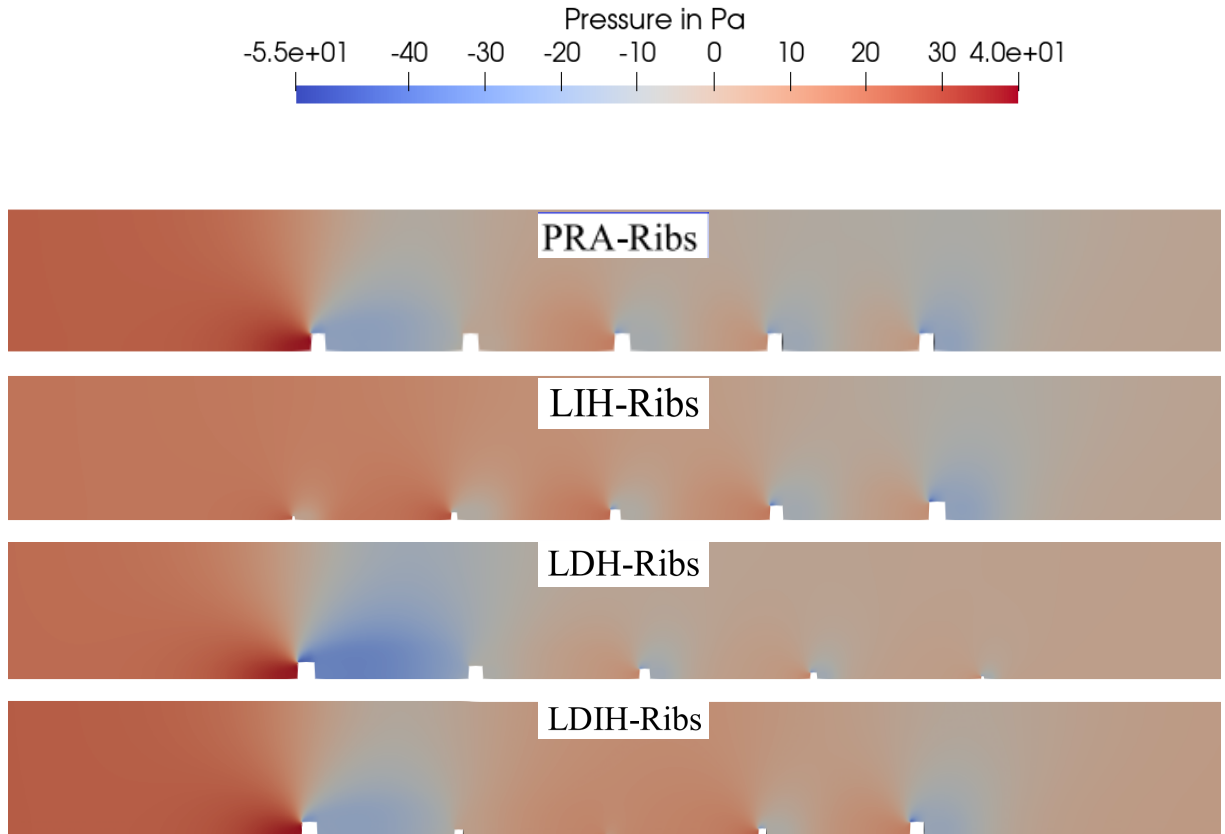


Figure 11: Contours of pressure for $Re = 80000$. The mainstream direction is left to right.

The pressure contours reveal that the most severe stagnation zones occur on the windward faces of the first two ribs, characterized by deep red high-pressure regions. Across all configurations, an immediate and sharp pressure drop (dark blue) is visible exactly at the leading upper edge of every rib, indicating the point where the freestream violently shears away from the solid boundary and accelerates.

Crucially, the contours physically demonstrate why the LDH (Linearly Decreasing Height) configuration successfully mitigates form drag. As the mainstream flows from left to right, the gradually lowering rib heights in the LDH setup allow the bulk fluid to ride smoothly over the obstacles without triggering massive, repeated stagnation pressure spikes on the downstream ribs, unlike the uniform baseline (PRA).

8.3 Convergence plots:

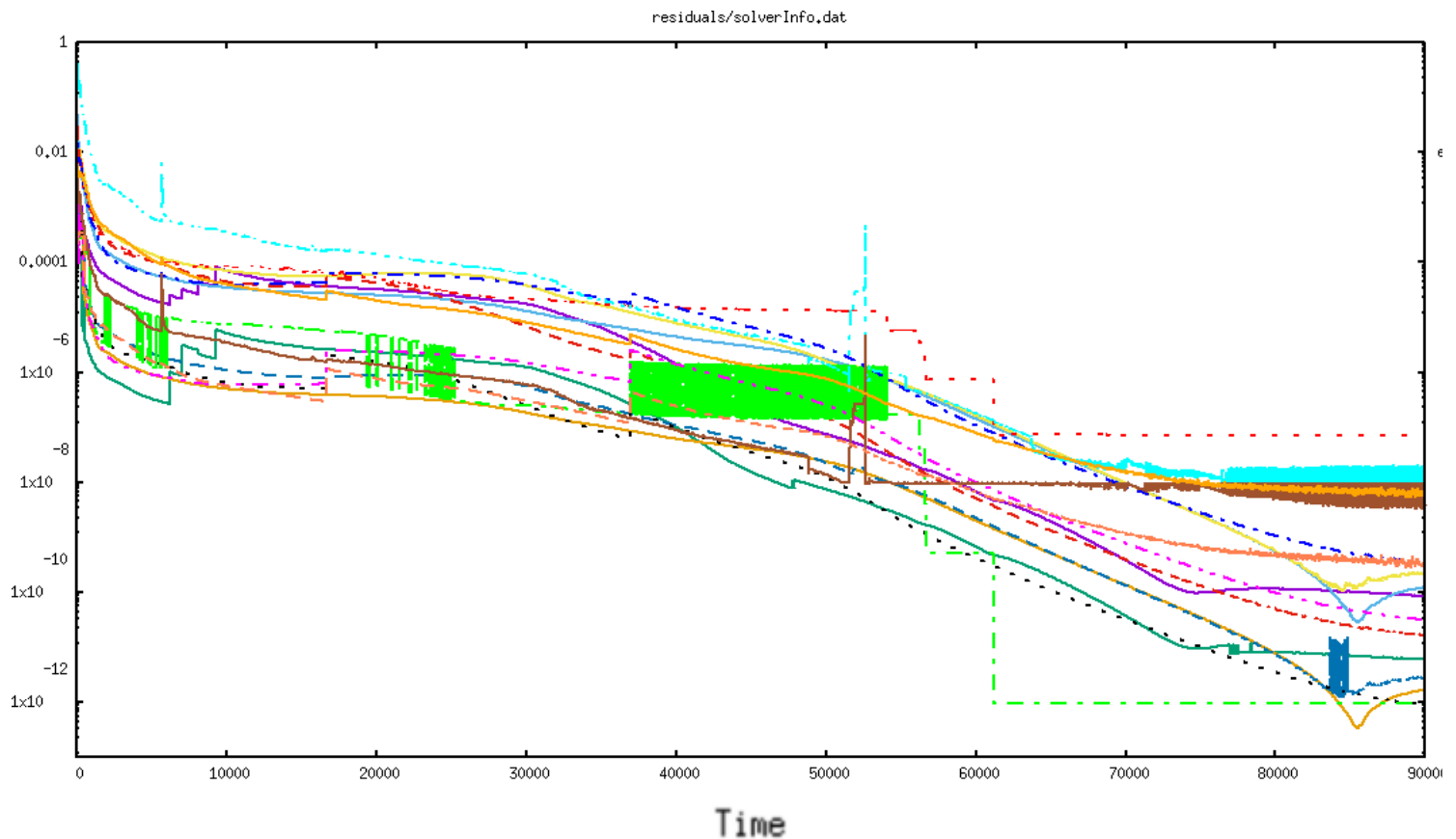


Figure 12: Residuals v/s Time plot for PRA ribs at $Re = 80000$.

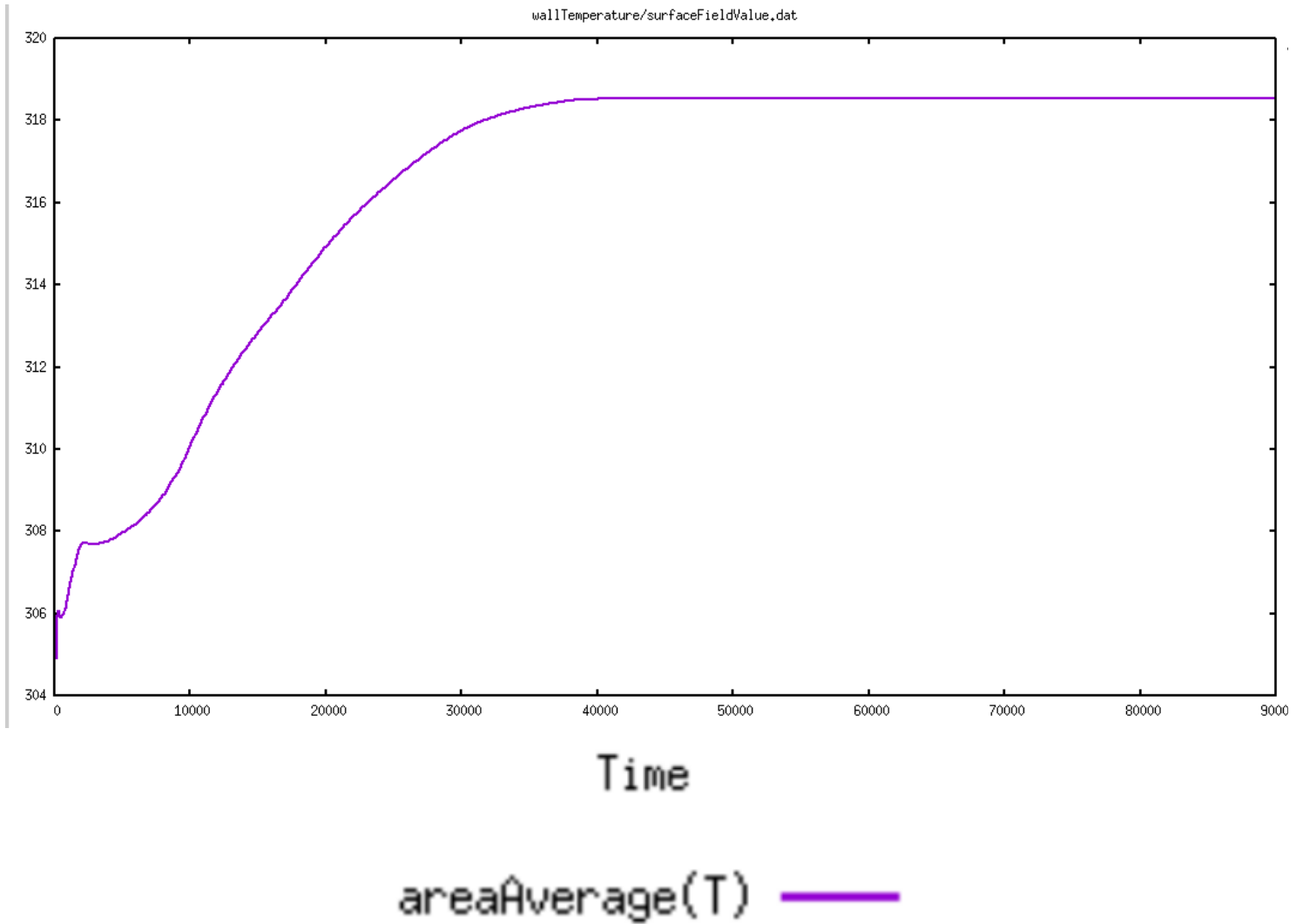


Figure 13: Plot of Area-averaged bottom wall temperature against Timesteps for PRA ribs geometry at $Re = 80000$.

The numerical stability and final accuracy of the steady-state OpenFOAM⁹ simulations were rigorously verified using a dual-criteria approach. To ensure complete convergence, both the mathematical solver residuals and the stabilization of a macroscopic physical parameter were monitored simultaneously.

Solver Residuals (Numerical Convergence)

The primary residual plot documents the iterative error decay for the governing equations, including momentum (U_x , U_y), pressure (p_{rgh}), turbulence quantities, and energy (T). During the initial phase of the simulation, the residuals exhibit expected transients as the base flow field establishes itself. As the solver progresses through the execution strategy—transitioning to higher-order spatial discretization schemes and tightened relaxation factors—the iterative errors

decay smoothly. The residuals for the momentum and turbulence equations successfully drop below the strict 10^{-7} threshold, while the energy equation achieves an exceptionally deep convergence. This mathematically validates that the linear matrix solvers have successfully converged.

Average Wall Temperature (Physical Convergence)

While low numerical residuals indicate matrix convergence, verifying a true steady state requires ensuring that the bulk flow physics have stopped changing. The second plot tracks the area-averaged temperature of the ribbed wall across the iteration count. The curve captures the initial thermal transient phase, where the wall temperature rapidly adjusts as the thermal boundary layer develops and the convective heat transfer mechanisms initialize. As the iterations continue, the curve asymptotically flattens into a perfectly horizontal line. This stabilization of the average wall temperature shows that the thermal field has reached equilibrium, and have physically converged.